Knights and Knaves

(Based on the book by R. Smullyan, *Satan, Cantor and Infinity*)

An explorer Kris Kolumba went to the island Kamelot. The island is inhabited by knights, who only make true statements, and knaves who only make false ones.

1. Kris ran into three people, Amelia, Benji, and Cyril. She asked Amelia, “Are Benji and Cyril both knights?” Amelia said, “Yes.” Kris then asked, “Is Benji a knight?” To her surprise, Amelia answered no. What were each of Amelia, Benji, and Cyril (knights or knaves)?
2. Trying to meet the island’s Sorceress Serena, Kris went to the house of her apprentice and sees three people. She asked, “Which one of you is the Sorcerer’s apprentice?” “I am,” replied one. “I am,” replied another. Kris then asked the third person, “can you tell me anything?” “It’s funny,” said the third, “At most, only one of the three of us will tell the truth. Which one was the apprentice? Was he a knight or a knave?
3. Impressed by Kris’ deductive skills the apprentice told her where the Sorceress was. When she went up to her home she saw two women, one wearing a blue hat, the other wearing a green one. Kris asked the woman in the blue hat, “Is the sorcerer a knight.” She answered yes or no and Kris deduced from this who was the sorcerer. What did the woman with the blue hat say and which person was the sorcerer? Did Kris know whether the sorcerer is a knight? If not what does she need to ask to find out?
4. After finding out that the sorcerer was a knight Kris asked her about her magic. Serena then disclosed that she does not use magic but rather logic to trick people. “In fact, ” she said, “I can ask you a yes or no question that will be logically impossible for you to answer correctly, even though the question has a correct answer.” What was the sorcerer’s question?
5. Another explorer Mark decided to travel to the island and met the sorcerer. He asked her how she had become interested in logic and she replied: “When I was young my Uncle visited the island. He ran into three people, Alfonso, Betty, and Carol. He asked Alfonso, ‘Are you a knight or a knave.’ Alfonso answered but in a foreign language. So he asked Betty what Alfonso had said and Betty replied, ‘Alfonso said he is a knave.’ Then Carol chimed in, ‘Don’t believe Betty, she’s lying.’ My uncle told me he figured out whether Carol was a knight or a knave, but had me figure out for myself.” Was Carol a knight or a knave?
6. Serena then recounted a story about an encounter she had had the previous week. “Just last week,” she said, I passed by someone on the island who I had never met before. As I passed him he muttered something, and because he muttered it the statement was false and I knew he was a knave.” What could the man have muttered? Notice that the sorcerer only new it was a lie because the man said it. Suppose the man had said something that he knew had to be true only because the man said it, and meant that the man was a knight. Then what could he have said?
7. Serena then told a story about her early travels on the island, “One day as I was going to the beach I reached a fork in the trail and didn’t know which way to go. There was a native there and I asked her a yes or no question and then knew the way.” What did the Sorcerer ask?
8. The next day while walking Mark met a pair of identical twins, Sally and Tessie, who were friends of the Sorcerer. He was told in advance that Tessie always lied and Sally always told the truth. He also knew that one of them was married. He then asked a three word, yes or no question, to one of them, and upon the answer knew whether Sally or Tessie was married. What did he ask? What could he ask if instead he wanted to know which twin was married (he doesn’t know which twin is which)?
9. Kris then returned to the island and became friends with the Chief Justice Joseph, a knight. Over dinner the judge said, “Yesterday I sentenced a knave to three months in jail for lying under oath. There were two defendants, Daphne and Eric. Daphne claimed that before the trial Eric told a friend ‘I lied yesterday.’ But Eric could not have said that because, a knight would never falsely say that he had lied, and a knave would never truthfully admit to lying.” Unfortunately Daphne was wrongly convicted, how so?
10. The next day Kris went to court with the Judge, where they were presented with a man named Fred, accused of stealing a watch. Kris asked the man, “Is it true that after the robbery you claimed you did not steal the watch?” Fred said, “Yes.” Kris then asked, “Did you ever claim you were the one who stole the watch.” Fred answered yes or no and then Kris knew whether he was innocent. What did Fred say and was he innocent?
11. The next case was about a stolen horse. There were four suspects, George, Hermione, Isaac, and Julie. The authorities were sure one of them was guilty and had brought the first three in for questioning, but they couldn’t find Julie so they were proceeding without her. The Judge asked the suspects, “Who stole the horse.” George replied, “Hermione stole it,” Hermione said, “Isaac stole it,” and Isaac said, “Julie stole it.” The one of the defendants said, “the other two are lying.” The Judge thought for a bit then pointed to one of the three and said “You are innocent and may leave.” The Judge then asked one of the remaining two if the other was a Knight, and after being told yes or no knew who was guilty. Who accused the other two of being guilty, who was acquitted, what was the answer to the last question, and who stole the horse?

1. The Judge next presided over a court case where he didn’t know whether the prosecutor or defense attorney’s were knights or knaves. The prosecutor claimed, “The defendant is guilty and has committed other crimes in the past.” The defense attorney claimed, “The defendant is innocent and the prosecutor is a knave.” The Judge then asked one of them a question he knew the answer to, so knew if that one was a knight or a knave. From this the Judge knew if the defendant was innocent or guilty. But if the other attorney had answered then we couldn’t know if the Judge would have known if the defendant was innocent or guilty. Who did the Judge ask, was the asked an attorney a knight or a knave, and was the man guilty?